Radiation Constrained Scheduling of Wireless Charging Tasks

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ABSTRACT

This paper studies the problem of Radiation cOnstrained scheduling of wireless Charging tasKs (ROCK), that is, given wireless charging tasks with required charging energy and charging deadline for rechargeable devices, scheduling the power of wireless chargers to maximize the overall effective charging energy for all rechargeable devices, and further to minimize the total charging time, while guaranteeing electromagnetic radiation (EMR) safety, *i.e.*, no point on the considered 2D area has EMR intensity exceeding a given threshold. To address ROCK, we first present a centralized algorithm. We transform ROCK from nonlinear problem to linear problem by applying two approaches of area discretization and solution regularization, and then propose a linear programming based greedy test algorithm to solve it. We also propose a distributed algorithm by presenting an area partition scheme and two approaches called area-scaling and EMR-scaling, and prove that it achieves effective charging energy no less than $(1-\varepsilon)$ of that of the optimal solution, and charging time no more than that of the optimal solution. We conduct both simulation and field experiments to validate our theoretical findings. The results show that our algorithm achieves 94.9% of the optimal effective charging energy and requires 47.1% smaller charging time compared with the optimal one when $\varepsilon \geq 0.2$, and outperforms the other algorithms by at least 350.1% in terms of charging time with even more effective charging energy.

CCS CONCEPTS

•Networks → Network control algorithms; •Theory of computation → Scheduling algorithms;

KEYWORDS

Wireless charging; EMR safety; Scheduling

1 INTRODUCTION

1.1 Motivation and Problem Statement

Recently, Wireless Power Transfer (WPT) technology is undergoing rapid development due to its advantages such as no contact, no wiring, reliable power supply, and ease of maintenance. So far, there are 215 companies, including IT leaders Qualcomm, Samsung, Philips, LG, and Huawei, have joined Wireless Power Consortium, an organization dedicated to promote standardization of WPT, and they have up to 848 registered WPT products [1]. By a recent report, wireless power transmission market is estimated to surge to 17.04 billions to 2020 [2].

In this paper, we study the problem of Radiation cOnstrained scheduling of wireless Charging tasKs (ROCK) with the optimization goal of maximizing the effective charging energy and further minimizing the charging time under the EMR safety constraint. A wireless charging task initiated by a rechargeable device consists of a required amount of charging energy (called *required charging energy*) and a time deadline for harvesting charging power (called *charging deadline*) for the device. As extra charging energy beyond the required charging energy for a device is useless, we define *effective charging energy* as the minimum value of the total charged energy before the charging deadline and the required charging energy. Suppose the emitting power of wireless chargers can be continuously adjusted from zero to a maximum value, and we can schedule the power of wireless chargers to handle the wireless charging tasks proposed by their covered rechargeable devices. We want to maximize the (overall) effective charging energy for all the devices, and further taking achieving maximum effective charging energy as a condition, we want to minimize the total charging time. Besides, as WPT technology commonly incurs high electromagnetic radiation (EMR), which causes risks of mental diseases, tissue impairment, brain tumor, miscarriage, and detrimental effect for children that can be even ten times greater than adults [3], we should guarantee EMR safety for our scheduling scheme. This indeed serves as the constraint for our optimization. To sum up, we state our problem ROCK as follows. Given a set of wireless chargers and rechargeable devices distributed in a 2D area and wireless charging tasks for the devices, schedule the power for all the chargers to maximize the effective charging energy and further to minimize the charging time while guaranteeing EMR safety, *i.e.*, no point on the area has EMR intensity exceeding a given threshold during the whole charging process.

1.2 Limitations of Prior Art

On one hand, there exist some works that study wireless charging but all of them overlook the EMR safety. Some of them [4–6] study charging efficiency issues in wireless charger networks where all chargers are static, but do not consider charging task scheduling. Others focus on mobile charging problems where one single or multiple chargers wander in a field to charge rechargeable devices deployed there to guarantee their normal working. However, their

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optimization performed for mobile chargers is typically from the perspectives such as path planning and charging time assignment for devices, which are fundamentally different from ours. On the other hand, other works [3, 7–10] consider the EMR safety in wireless charger networks, but none of them consider charging task scheduling.

1.3 Key Technical Challenges

The first challenge is that ROCK is nonlinear. The EMR safety requirement is imposed on every point on the plane which indicates an infinite number of constraints; the effective charging energy evaluation function is nonlinear; the chargers' adjusting factors can continuously and arbitrarily change over time which typically requires solving Hamilton–Jacobi–Bellman partial differential equations [11].

The second challenge is that even if we approximately convert the problem of maximizing the effective charging energy to a simple linear program, ROCK falls into the realm of classical Quadratically Constrained Linear Programming with non-positive semidefinite constraint matrix, which invalidates traditional convex optimization approaches.

The third challenge is to design a distributed algorithm scalable with network size for ROCK. As neighboring chargers may cover the same devices and their EMR coverage area may overlap, their charging energy and incurred EMR couple with others, which inherently requires global optimization for the network as a whole and therefore inhibits designing distributed scalable algorithms.

The fourth challenge is to simultaneously bound the effective charging energy and charging time for the distributed algorithm. First, the relationship between maximum effective charging energy and minimum charging time is complicated, *e.g.*, the minimum charging time may be prolonged or shortened when the maximum effective charging energy is increased if the constraints for ROCK are relaxed. Second, when we convert ROCK from nonlinear problem to linear problem and when we decompose ROCK to make it distributed, both the optimization goal and constraints for ROCK inevitably change, so do the maximum effective charging energy and the minimum charging time, which is quite complicated.

1.4 Proposed Approaches

We propose a centralized algorithm and a distributed algorithm for ROCK. For the centralized version, we first drop the optimization goal of charging time and study the relaxed version of ROCK, *i.e.*, scheduling charging tasks to maximize the effective charging energy. We apply an *area discretization* technique to approximate the continuous and nonlinear EMR safety constraint as a finite number of linear constraints. Then, we propose an approach called *solution regularization* to map any arbitrary solution to a piecewise constant solution without performance loss, which dramatically reduces the solution space. Finally, we transform the problem to a linear programming problem that can be easily addressed. We thereby address the first challenge. Further, to deal with ROCK, we leverage the monotonicity of the effective charging energy with respect to the charging time, and propose a *linear programming based greedy test algorithm*, which not only yields the optimal result but also has fast convergence speed. We thus address the second challenge.

For the distributed version, we first propose an area partition scheme to partition the whole area into many subareas, and switch off the chargers lying on the boundaries of the subareas to eliminate the impact of charging power and EMR from the surrounding subareas. Then, we can safely consider each subarea independently. Further, to bound the performance loss of effective charging energy, we enumerate a fixed number of partition schemes rather than apply one specific area partition scheme. We then forge a solution for each charger by reasonably synthesizing the obtained solutions for the partition schemes so that the resulted global solution must be also feasible and, more importantly, each charger only needs information from other chargers within a constant distance. Therefore, we address the third challenge. Further, to bound the overall effective charging energy and charging time, we first propose an approach called *area-scaling* to find a suitable target effective charging energy, rather than the maximum effective charging energy, to decouple the complex relationship between the achieved effective charging energy and charging time to some extent. Then, we propose an approach called *EMR-scaling* to artificially adjust the EMR constraints and the solution. By constructing a series of transient problems with suitable optimization targets and constraints and guaranteeing that the performance gap for the solutions to any pair of adjacent problems in the series can be evaluated and bounded, we prove that the ultimate solution is feasible, and achieves overall effective charging energy no less than $(1-\varepsilon)$ of that of the optimal solution and charging time no more than that of the optimal solution. Then, we address the fourth challenge.

1.5 Evaluation Results

The simulation results show that our proposed distributed algorithm to ROCK achieves at least 94.9% of the optimal effective charging energy and 47.1% smaller charging time compared with the optimal when $\varepsilon \geq 0.2$, outperforms the other algorithms by at least 2.0% in terms of effective charging energy and 350.1% in terms of charging time, and its network delay approaches a constant as network size scales up. The field experimental results show that our algorithm requires only 27.2% $\sim$ 85.5% of the charging time for compared algorithms given the same target effective charging energy.

1.6 Overview

The remainder of the paper is organized as follows. Section 2 briefly reviews the related work, and Section 3 formally states the ROCK problem. Before addressing ROCK in Section 5, Section 4 considers a relaxed version of ROCK, *i.e.*, charging task scheduling with maximum charging energy. Next, Section 6 proposes a distributed algorithm for ROCK. Section 7 and Section 8 present simulation results and experimental results, respectively, and Section 9 concludes the paper.

2 RELATED WORK

First, there exist some works that study wireless charging but overlook the EMR safety. On one hand, some of them [4–6] focus on charging efficiency issues in wireless charger networks where all chargers are static, with no regard for charging task scheduling. For example, we presented the directional charging problem where both the charging area for chargers and receiving area for devices can be modeled as sectors, and studied ominidirectional charging using directional chargers in [5]. On the other hand, others concentrate on mobile charging scenarios where one single or multiple chargers wander in a field of interest to charge rechargeable devices deployed there to ensure their normal working; nevertheless, their optimization performed for mobile chargers is typically from the perspectives such as path planning and charging time assignment for devices, which are fundamentally different from that in this paper. Particularly, [12–18] are concerned with charging efficiency of chargers. [19–21] target on optimizing the service delay of mobile chargers. [22–29] focus on network performance issues such as data collection, data routing, event monitoring, and task assignment.

Second, some other works [3, 7–10] consider the EMR safety in wireless charger networks by guaranteeing that the EMR intensity at any point in the considered area should not exceed a predefined EMR threshold, but none of them consider charging task scheduling. For example, we presented the safe charging problem that considers EMR safety during charging, and studied how to schedule non-adjustable chargers [3] and adjustable chargers [7] to maximize overall charging utility.

3 PROBLEM STATEMENT

Suppose we have n static wireless chargers $S = \{s_1, s_2, \ldots, s_n\}$ together with m static rechargeable devices $O = \{o_1, o_2, \ldots, o_m\}$ located on a 2D plane Ω. All wireless chargers are identical and equipped with omnidirectional antennas to charge devices via wireless.

We adopt the charging model with adjustable power proposed in [7]. Suppose the distance between a charger and a device is d, and chargers can continuously adjust their emitting power from zero to a maximum value, *i.e.*

$$P(d) = \begin{cases} \frac{\alpha}{(d+\beta)^2}, & d \le D \\ 0, & d > D \end{cases} \tag{1}$$

where α and β are known constants determined by hardware parameters of chargers and devices and surrounding environment, and D is the farthest distance the charging power can reach. We define adjusting factor x as the ratio of the adjusted power to the maximum power ($0 \le x \le 1$), the received power by the device from the charger is then $P(d)x$. Moreover, we assume the wireless power received by a device from multiple chargers is additive [3, 4, 7]. Let $d(s_i, p)$ denote the distance between s_i and a point p on the plane Ω and x_i denote the adjusting factor of charger s_i, then the charging power there can be expressed as $\sum_{s_i \in S} P(d(s_i, p))x_i$. Table 1 lists the notations and symbols used in this paper.

We adopt the widely accepted EMR model proposed in [3, 7]. The EMR value $e(p)$ at a position p is proportional to the charging power there, and it is also additive, *i.e.*

$$e(p) = \sum_{s_i \in S} e(d(s_i, p)) = C_1 \sum_{s_i \in S} P(d(s_i, p))x_i. \tag{2}$$

where C_1 is a predetermined constant.

Table 1: Notations and symbols used in this paper

Symbol	Description
s_i	*i-th* wireless charger
o_j	*j-th* rechargeable device
n	Number of wireless chargers
m	Number of rechargeable devices
α, β	Constants in the charging model
D	Farthest charging distance for chargers
E_j	Required charging energy of device o_j
T_j	Charging deadline of device o_j
R_t	EMR threshold
$x_{i,k}$	Adjusting factor of the *i-th* charger at the *k-th* scheduling round
$\widetilde{e}_{iz}$	Approximated EMR in subarea $\mathcal{A}_z$ from charger s_i

Suppose device o_j has a required charging energy of E_j, and a charging deadline of T_j. Without loss of generality, we assume that T_j ($j = 1, \ldots, m$) is sorted in ascending order. It desires that upon the charging deadline, it could receive an amount of charging energy no less than E_j; if it is impossible, then the larger the better. We define *effective charging energy* to capture the usefulness of the received energy of device o_j, which can be formally expressed as $\min\{E'_j, E_j\}$ where E'_j is the amount of actual received energy of o_j till time point T_j which can be greater or less than E_j. Note that device o_j may be still be charged after its deadline T_j, but such additional amount of energy is useless for o_j. Whenever a device o_j needs to be charged, it sends a charging request including the required charging energy as well as the charging deadline to its surrounding chargers covering it. The charging task information in the request is then used by the centralized algorithm or the distributed algorithm for ROCK.

Formally, we state our problem as follows. *Given a set of wireless chargers S and rechargeable devices O, and each device o_j has a wireless charging task with required charging energy of E_j and charging deadline of T_j, scheduling the power of all chargers so that the overall effective charging energy is maximized and further the charging time is minimized, and no point on the 2D plane Ω has EMR intensity exceeding a given threshold R_t during the charging process.*

4 CHARGING TASK SCHEDULING WITH MAXIMUM EFFECTIVE CHARGING ENERGY

In this section, we consider a relaxed version of ROCK (ROCK-R for short), *i.e.*, charging task scheduling with maximum charging energy, to pave the way to address ROCK. We first formulate ROCK-R, and then apply an area discretization technique to approximate the continuous and nonlinear EMR safety constraint as a set of linear constraints. Next, we propose a regularization method to confine the solution space. Last, we equivalently transform the formulated problem to a linear program that can be easily addressed.

4.1 Problem Formulation for ROCK-R

As any charger can adjust its power continuously over time and independently of each other, we denote by $x_i(t)$ the function of adjusting factor x_i with time t ($t \in [0, T_m]$ is the time duration of the whole scheduling algorithm) for charger s_i. To satisfy the EMR safety constraint, we require

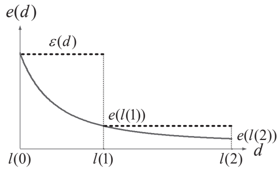


Figure 1: EMR approximation

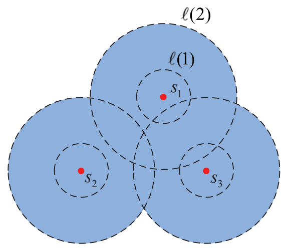


Figure 2: Area discretization

that the EMR intensity at any point p on the plane Ω, *i.e.* $\sum_{i=1}^{n} C_1[P(d(s_i,p))x_i(t)]$, should not exceed a given EMR threshold R_t at any time point t ($t \in [0, T_m]$), which indicates $C_1 \sum_{i=1}^{n}[P(d(s_i,p))x_i(t)] \leq R_t$. Then, the effective charging energy is $\min\{\int_0^{T_j} \sum_{i=1}^{n}[P(d(s_i,o_j))x_i(t)]dt, E_j\}$. Our goal is to maximize the overall effective charging energy for all devices, which can be written as $\max \sum_{j=1}^{m} \min\{\int_0^{T_j} \sum_{i=1}^{n}[P(d(s_i,o_j))x_i(t)]dt, E_j\}$. Then, we can formulate the ROCK-R problem as follows.

$$
\begin{aligned}
&\textbf{(P1)} \quad \max_{x_i(t)} \sum_{j=1}^{m} \min\{\int_0^{T_j} \sum_{i=1}^{n}[P(d(s_i,o_j))x_i(t)]dt, E_j\} \\
&\quad s.t. \quad C_1 \sum_{i=1}^{n}[P(d(s_i,p))x_i(t)] \leq R_t, (\forall p \in \Omega; t \in [0,T_m]) \\
&\quad\quad\quad 0 \leq x_i(t) \leq 1. \quad (i = 1,\ldots,n) \qquad (3)
\end{aligned}
$$

Note that we need to compute $x_i(t)$ $(i = 1,\ldots,n)$ in the optimization problem **P1**.

4.2 Area Discretization

First, we use a piecewise constant function to approximate the EMR function $e(d)$ as shown in Figure 1. Suppose the endpoints of the piecewise constant line segments are $\ell(1),...,\ell(Q)$ ($\ell(0) = 0, \ell(Q) = D$) in order. Accordingly, we draw concentric circles for a charger with radius $\ell(1),...,\ell(Q)$, respectively. The whole plane is then partitioned into multiple subareas which are shaped by these concentric circles. As the approximated EMR from each charger is constant within the same subarea, the aggregated approximated EMR, which is the sum of all approximated EMR from all chargers, is also constant within the same subarea. Figure 2 shows an instance where the endpoints for piecewise constant segments are $\ell(1)$ and $\ell(2)$, and therefore 2 concentric circles are drawn for 3 chargers, which results in 12 subareas. We have the following theorem to describe the EMR approximation error.

THEOREM 4.1. *[7] Setting $\ell(0) = 0$, $\ell(Q) = D$, and $\ell(q) = \beta((1+\varepsilon)^{q/2} - 1)$, $(q = 1,...,Q-1)$, and using the following piecewise constant function $\widetilde{e}(d)$*

$$
\widetilde{e}(d) = \begin{cases} e(\ell(0)), & d = \ell(0) \\ e(\ell(q-1)), & \ell(q-1) < d \leq \ell(q)\ (q = 1,...Q) \\ 0, d > D. \end{cases} \qquad (4)
$$

where ε is a given error threshold, the EMR approximation error for any position p in a certain subarea satisfies

$$
1 \leq \frac{\widetilde{e}(p)}{e(p)} \leq 1 + \varepsilon. \qquad (5)
$$

After all, the problem **(P1)** can be rewritten as:

$$
\begin{aligned}
&\textbf{(P2)} \quad \max_{x_i(t)} \sum_{j=1}^{m} \min\{\int_0^{T_j} \sum_{i=1}^{n} P_{ij}x_i(t)dt, E_j\} \\
&\quad s.t. \quad \sum_{i=1}^{n} \widetilde{e}_{iz}x_i(t) \leq R_t, (z = 1,\ldots,Z; t \in [0,T_m]) \\
&\quad\quad\quad 0 \leq x_i(t) \leq 1. \quad (i = 1,\ldots,n) \qquad (6)
\end{aligned}
$$

where P_{ij} is the abbreviated form of $P(d(s_i,o_j))$, $\widetilde{e}_{iz}$ is the approximated EMR in subarea $\mathcal{A}_z$ from charger s_i, and Z is the number of all subareas. Our goal is to determine $x_i(t)$ $(i = 1,\ldots,n)$ for all chargers.

4.3 Solution Regularization

One key technical challenge of **P2** is due to the arbitrariness of chargers' adjusting factors, which can continuously change over time. Generally, we need to solve Hamilton–Jacobi–Bellman partial differential equations [11] when dealing with continuous time control problems, which is typically very challenging. To address this challenge, we present an approach called solution regularization. We first give its formal definition.

Definition 4.1. ***(Solution Regularization)*** *For any feasible solution $x_i(t)$ $(i = 1,\ldots,n)$ to problem **P2**, its regularized form is defined as $x_{i,k} = \frac{\int_{T_{k-1}}^{T_k} x_i(t)}{T_k - T_{k-1}}$ $(k = 1,\ldots,m)$ during the time period $[T_{k-1}\ T_k)$ for $k = 1,\ldots,m-1$ and $[T_{m-1}\ T_m]$ where $T_0 = 0$.*

LEMMA 4.2. *For any feasible solution for ROCK-R, its regularized form is still feasible and achieves the same effective charging energy.*

PROOF. We refer readers to [30] for more details. □

As any feasible solution is equivalent to its regularized form by Lemma 4.2, we only need to consider solutions in regularized forms. Then, ROCK-R can be rewritten as

$$
\begin{aligned}
&\textbf{(P3)} \quad \max_{x_{i,k}} \sum_{j=1}^{m} \min\{\sum_{i=1}^{n} P_{ij} \sum_{k=1}^{j} x_{i,k}(T_k - T_{k-1}), E_j\} \\
&\quad s.t. \quad \sum_{i=1}^{n} \widetilde{e}_{iz}x_{i,k} \leq R_t, (z = 1,\ldots,Z; k = 1,\ldots,m) \\
&\quad\quad\quad 0 \leq x_{i,k} \leq 1. \quad (i = 1,\ldots,n; k = 1,\ldots,m) \qquad (7)
\end{aligned}
$$

where P_{ij} is the abbreviated form of $P(d(s_i,o_j))$, and $\widetilde{e}_{iz}$ is the approximated power in subarea $\mathcal{A}_z$ from charger s_i. Note that $x_{i,k}$s $(i = 1,\ldots,n; k = 1,\ldots,m)$ are the decision variables.

4.4 Problem Transformation

Problem **P3** cannot be straightforwardly addressed, therefore we transform it to a linear programming problem. We introduce assistant variables y_j, and rewrite **P3** as follows:

$$
\begin{aligned}
&\textbf{(P4)} \quad \max_{x_{i,k}, y_j} \quad \sum_{j=1}^{m} y_j \\
&\quad s.t. \quad \sum_{i=1}^{n} \widetilde{e}_{iz}x_{i,k} \leq R_t, (z = 1,\ldots,Z; k = 1,\ldots,m) \\
&\quad\quad\quad y_j \leq \sum_{i=1}^{n} P_{ij} \sum_{k=1}^{j}(T_k - T_{k-1})x_{i,k}, \\
&\quad\quad\quad y_j \leq E_j, \quad (j = 1,\ldots,m) \\
&\quad\quad\quad 0 \leq x_{i,k} \leq 1. \quad (i = 1,\ldots,n; k = 1,\ldots,m). \qquad (8)
\end{aligned}
$$

Note that $x_{i,k}$s $(i = 1,\ldots,n; k = 1,\ldots,m)$ and y_j $(j = 1,\ldots,m)$ are the decision variables.

LEMMA 4.3. *The optimal solution of $x_{i,k}$ for problem **P4** is also optimal to problem **P3**, and they achieve the same optimal objective value.*

Proof. We refer readers to [30] for more details. □

Apparently, problem **P4** is a linear program, thus we use LINGO to address problem **P4**. The following theorem indicates the performance of our algorithm to ROCK-R.

Theorem 4.4. *Our algorithm to ROCK-R achieves* $\frac{1}{1+\varepsilon}$ *approximation ratio for any* $\varepsilon > 0$*, and its time complexity is* $O(m^{4.5}n^{6.5}\varepsilon^{-2})$ *where* m *and* n *are the numbers of the rechargeable devices and the wireless chargers, respectively.*

Proof. We refer readers to [30] for more details. □

5 CENTRALIZED ALGORITHM FOR ROCK

Suppose the achieved optimal overall effective charging energy for problem **P4** is E_{max}. ROCK requires that the overall effective charging energy $\sum_{j=1}^{m} y_j$ must achieve E_{max}, and meanwhile the charging time t is minimized. In addition, as the charging time t becomes a variable, it can be of any size and fall between two adjacent deadlines, say $T_{m'-1}$ and $T_{m'}$. Then, we have $T_{m'-1} < t \le T_{m'}$. For device o_j, if t is greater than its deadline T_j, then its overall charging energy until T_j is $\sum_{i=1}^{n} P_{ij} \sum_{k=1}^{j}(T_k - T_{k-1})x_{i,k}$; otherwise, o_j will be charged until time t, and its overall charging energy becomes $\sum_{i=1}^{n} P_{ij}[\sum_{k=1}^{m'-1}(T_k - T_{k-1})x_{i,k} + (t - T_{m'-1})x_i^{m'}]$.

Therefore, ROCK can be formulated as follows.

$$
\begin{aligned}
\textbf{(P5)} \quad & \min \quad t \\
s.t. \quad & T_{m'-1} < t \le T_{m'}, \\
& \sum_{i=1}^{n} \widetilde{e}_{iz} x_{i,k} \le R_t, (z = 1, \ldots, Z; k = 1, \ldots, m') \\
& y_j \le \sum_{i=1}^{n} P_{ij}[\sum_{k=1}^{m'-1}(T_k - T_{k-1})x_{i,k} + (t - T_{m'-1})x_i^{m'}], \\
& y_j \le \sum_{i=1}^{n} P_{ij} \sum_{k=1}^{j}(T_k - T_{k-1})x_{i,k}, \\
& y_j \le E_j, \quad (j = 1, \ldots, m), \\
& \sum_{j=1}^{m} y_j = E_{max}, \\
& 0 \le x_{i,k} \le 1. \quad (i = 1, \ldots, n; k = 1, \ldots, m).
\end{aligned} \tag{9}
$$

Unfortunately, the third constraint is quadratic as it contains quadratic terms $tx_i^{m'}$, and thus ROCK falls in the realm of Quadratically Constrained Linear Programming (QCLP), which is generally NP-hard [31]. Further, the constraint matrix for ROCK is not positive semidefinite, and therefore no traditional convex optimization techniques can be employed for an optimal solution [32]. One way is to use semidefinite relaxation to obtain a non-optimal solution with loose performance bound [33], and another is to use solvers to numerically solve this problem, *e.g.*, we have tested several nonlinear solvers such as Gurobi, Cplex, Nlopt, and Snopt, Knitro, Conopt, Stoaminlp, Minlpsolve, Strustr included in Tomlab, and found that even the best one, Conopt [34], in terms of failure rate, accuracy, and time efficiency, have failure rate of at least 66.7% and running time being hundreds of times that of ours.

We propose a linear programming based greedy test algorithm (LP-GTA) as shown in Algorithm 1 to optimally address ROCK. The key idea of LP-GTA is to exploit the monotonicity of the effective charging energy with respect to the charging time t. It sets the whole charging time as t, which means it adjusts the deadlines of devices in problem **P4** so that all deadlines of devices are no more than t, and then, by trying different value of t using a binary search method and solving the linear program **P4**, finds the minimum t while the effective charging energy is maximized.

Algorithm 1: Linear Programming based Greedy Test Algorithm

Input: Required charging energy E_j, charging deadline T_j, EMR threshold R_t, approximated power $\widetilde{e}_{iz}$ in subarea $\mathcal{A}_z$ from charger s_i, terminating threshold ε_T, and optimal effective charging energy E_{max}

Output: Adjusting factor $x_{i,k}$ and charging time t

```
t_l = 0, t_r = T_m;
while 2(t_r − t_l)/(t_r + t_l) > ε_T do
    Set t' = ⌊(t_l + t_r)/2⌋ for problem P4;
    for All deadlines T_j s for devices in problem P4 do
        if T_j > t' then
            T_j = t';
    Solve P4 to get an optimal solution E'_max;
    if E'_max = E_max then
        t_r = t';
    else
        t_l = t';
Set t = (t_r + t_l)/2 and reset all deadlines T_j s that are greater
    than t as t; solve P4 and output the obtained adjusting factor
    x_{i,k} and charging time t.
```

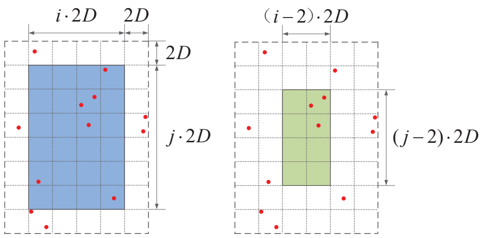
6 DISTRIBUTED ALGORITHM FOR ROCK

In this section, we develop a fully distributed algorithm for ROCK. We first give the key intuitions of the algorithm, and then describe details and analyse its performance.

6.1 Key Intuitions

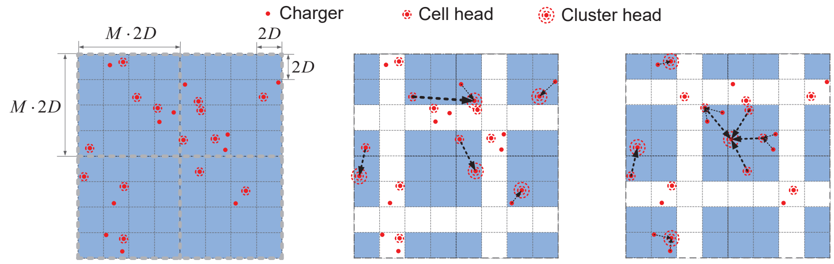
First, to degrade global computation to local ones, we propose an area partition scheme to partition the whole area into many subareas, and switch off the chargers lying on the boundaries of the subareas to eliminate the impact of charging power and EMR from the surrounding subareas. Thus, we can safely consider each subarea independently in a distributed manner. Further, to help bound the performance of effective charging energy, we enumerate a fixed number of partition schemes rather than apply one specific area partition scheme. As each charger is located in different subareas for different partition schemes which lead to different solutions, we forge a solution for each charger by reasonably synthesizing the obtained solutions so that the resulted global solution must be also feasible. This framework for distributed computing only needs information from chargers within a constant distance.

Second, we need to bound the overall effective charging energy and charging time for our distributed algorithm. By the above area partition scheme, the chargers in a subarea can charge the devices inside the subarea to a maximum effective charging energy that is even higher than that of the optimal solution to **P3** because the EMR from chargers outside the subarea is eliminated and thus the EMR constraint is relaxed. However, this may lead to a charging time that may be longer (as the maximum effective charging energy is



(a) $(M-1)$-*Cluster* (b) Inner rectangle

Figure 3: Area-scaling illustration



(a) (b) (c)

Figure 4: M-*Cluster*s**, $(M-1)$-*Cluster*s for $<3,2>$ and $<2,3>$**

higher) or shorter (as the EMR constraint is relaxed) than the optimal solution to **P3**. Consequently, rather than pursuing the maximum effective charging energy, we propose an approach called *area-scaling* to find a suitable target effective charging energy to decouple the complicated relationship between effective charging energy and charging time. Besides, we also need to bound the charging time gap between our solution under the setting of **P3** with finite approximated EMR constraints and the optimal solution to the original problem **P1** with infinite continuous EMR constraints, as well as that between the distributed algorithm and centralized algorithm. We propose an approach called *EMR-scaling* to artificially adjust the EMR constraints as well as the computed solution. Thereby, we can bound the effective charging energy and charging time by constructing a series of transient problems with suitable optimization targets and constraints and guaranteeing that the charging time gap for the solutions to any pair of adjacent problems in the series can be evaluated and bounded in theoretical analysis.

6.2 Algorithm Description

Algorithm 2 describes the details of the algorithm. First, it partitions the considered area into $2D \times 2D$ geographical *cell*s where D is the farthest charging distance for chargers, and further groups $M \times M$ such cells in to larger M-*Cluster*s ($M = \lceil \frac{3}{1-\sqrt{1-\varepsilon/3}} \rceil$). Without loss of generality, we assume that we have an integral number of M-*Cluster*s since otherwise we can introduce phantom cells with no chargers to achieve this goal. Each charger identifies itself as a member of a certain cell as well as the corresponding M-*Cluster* in a distributed manner with its geographical location information (Step 3). Further, chargers in the same cell elect a *cell head* using algorithms such as voting to handle information collection and dissemination, and computation tasks (Step 4). Figure 4(a) shows an instance of the process of Algorithm 2. The whole area is divided into 64 cells which further form 4 M-*Cluster*s. Each black dot denotes a charger, and each black dot surrounded by a dashed circle denotes a cell head.

Second, the algorithm adopts a so-called *turn-off* policy to further partition the area. We define turn-off policy of M-*Cluster*s as a tuple of $<p,q>$ by adopting which a M-*Cluster* turns off all the chargers located at the cells lying in the p-th row and q-th column of the M-*Cluster*. By letting all M-*Cluster*s employ the same turn-off policy, the whole area would be partitioned into a number of new clusters having size no more than $(M-1)\cdot 2D \times (M-1)\cdot 2D$ and

Algorithm 2: Distributed Algorithm for ROCK at Charger s_i

Input: Required charging energy E_j, charging deadline of T_j, EMR threshold R_t, and error threshold ε
Output: Adjusting factor $x_{i,k}$

1 Set the error threshold for EMR approximation as $\varepsilon/3$, and obtain the approximated power $\widetilde{e}_{iz}$ in each subarea $\mathcal{A}_z$ from each charger s_i;
2 Set $M = \lceil \frac{3}{1-\sqrt{1-\varepsilon/3}} \rceil$;
3 Classify itself into a cell based on its geographical information;
4 Elect a cell head in its cell;
5 **if** *It is a cell head* **then**
6 Elect cluster heads for all $(M-1)$-*Cluster*s for different turn-off policies that cover it;
7 **for** *All $(M-1)$-Clusters for different turn-off policies that cover it* **do**
8   **if** *It is a cluster head* **then**
9   Receive related information from all cell heads in the $(M-1)$-*Cluster*;
10   Use linear programming approaches to address the formed linear programming problem with the information from the chargers and devices in the rectangle located at the center of the considered $(M-1)$-*Cluster* with both length and width being $2D$ smaller, and obtain the optimal overall effective charging energy E^I_{max}; ▷ Area-scaling
11   Use the linear programming based greedy test algorithm presented in Algorithm 1 and set the input optimal overall effective charging energy as E^I_{max} and the EMR threshold as $\frac{1}{1-\varepsilon/3}R_t$, and compute a solution; ▷ EMR-scaling
12   Reduce the adjusting factors for all chargers in the solution to its $(1-\varepsilon/3)$; ▷ EMR-scaling
13   Send the solution to all the cell heads;
14   **else**
15   Send related parameters to the corresponding cluster head, and receive the adjusting factors for the chargers in its cell from the cluster head;
16 Send the related adjusting factor to all chargers in its cell;
17 **else**
18 Send related parameters to the cell head, and receive $M \times M$ adjusting factor results from the cell head in its cell;
19 Output the average value of the obtained $M \times M$ adjusting factors.

we call them $(M-1)$-*Cluster*s. Cell heads located in the same $(M-1)$-*Cluster* elect one among them as *cluster head* to take charge of the computation task of the chargers in the $(M-1)$-*Cluster* (Step 6). For different turn-off policies, the formed $(M-1)$-*Cluster*s are also different and the elected cluster heads can be also different to balance computation overhead across cell heads. Figure 4(b) shows the obtained 9 $(M-1)$-*Cluster*s when all M-*Cluster*s adopt the turn-off policy $<3,2>$. Each black dot surrounded by two dashed circles denotes an elected cluster head. The thin and thick dashed arrows respectively indicate the reporting from chargers to their cell head and from cell heads to their cluster head. Figure 4(c) shows the case for the turn-off policy $< 2, 3 >$.

Third, the algorithm applies the area-scaling approach in each $(M-1)$-*Cluster* (Step 10). First, the cluster head in the considered $(M-1)$-*Cluster* computes the maximum effective charging energy for chargers inside the rectangle located at the center of the considered $(M-1)$-*Cluster* with both length and width being $2D$ smaller than that of the $(M-1)$-*Cluster*, *e.g.*, the square with width of $(i-2)\cdot 2D$ and height of $(j-2)\cdot 2D$ as shown in green in Figure 3(b) for the $(M-1)$-*Cluster* with width of $i \cdot 2D$ and height of $j \cdot 2D$ as shown in Figure 3(a). Then, it uses the obtained result as the target effective charging energy, and applies the EMR-scaling approach by relaxing the EMR threshold to $\frac{1}{1-\varepsilon/3}R_t$ (Step 11), employing the linear programming based greedy test algorithm to compute the optimal solution, and then reducing all the obtained adjusting factors to its $(1-\varepsilon/3)$ (Step 12). After that, it disseminates the solution to all cell heads in the $(M-1)$-*Cluster* (Step 13). Each $(M-1)$-*Cluster* can independently compute its solution because the $2D$ distance between adjacent $(M-1)$-*Cluster*s guarantees no overlap of charging power or EMR from other $(M-1)$-*Cluster*s.

Fourth, the algorithm repeats the above process for all $M \times M$ different turn-off policies, and accordingly each charger will obtain $M \times M$ adjusting factors (Step 18). Then, each charger takes the average value of the $M \times M$ adjusting factors as the output (Step 19).

6.3 Performance Analysis

We have the following theoretical result for Algorithm 2.

Theorem 6.1. The output of Algorithm 2 for ROCK is a feasible solution. Its achieved effective charging energy is no less than $(1-\varepsilon)$ of that of the optimal solution, and charging time is no more than that of the optimal solution. Further, the communication complexity of Algorithm 2 is $O(\varepsilon^{-1})$.

Proof. For convenience, define the solution (vector) for charger s_i as $\mathbf{x}_i \triangleq (x_{i,1}, \ldots, x_{i,m})$, and the solution (vector) for all chargers as $\boldsymbol{\pi} \triangleq (\mathbf{x}_1, \ldots, \mathbf{x}_n)$. Suppose the obtained solution for the turn-off policy $< p, q >$ in Algorithm 2 is $\boldsymbol{\pi}^{<p,q>} = (\mathbf{x}_1^{<p,q>}, \ldots, \mathbf{x}_n^{<p,q>})$ where $\mathbf{x}_i^{<p,q>} = (x_{i,1}^{<p,q>}, \ldots, x_{i,m}^{<p,q>})$, and the solution obtained at Step 11 in Algorithm 2 with a "relaxed" EMR threshold $\frac{1}{1-\varepsilon/3}R_t$ is $\boldsymbol{\pi}_R^{<p,q>}$. Obviously, we have $\sum_{i=1}^n \widetilde{e}_{iz} x_{i,k}^{<p,q>^R} \le \frac{1}{1-\varepsilon/3}R_t$. As the final solution $\boldsymbol{\pi}^{<p,q>}$ is equal to $(1-\varepsilon/3)\boldsymbol{\pi}_R^{<p,q>}$ as per Step 12 in Algorithm 2, we get

$$\begin{aligned}\sum\nolimits_{i=1}^n \widetilde{e}_{iz} x_{i,k}^{<p,q>} &= \sum\nolimits_{i=1}^n (1-\varepsilon/3)\widetilde{e}_{iz} x_{i,k}^{<p,q>^R} \\ &\le (1-\varepsilon/3)\frac{1}{1-\varepsilon/3}R_t = R_t, \end{aligned} \tag{10}$$

which indicates $\boldsymbol{\pi}^{<p,q>}$ is feasible for $z \in [1,\ Z]$ and $p, q \in [1,\ M]$. Moreover, the output solution of Algorithm 2 is $\overline{\boldsymbol{\pi}} = \sum_{p=1}^M \sum_{q=1}^M \boldsymbol{\pi}^{<p,q>}/M^2$, we get

$$\sum\nolimits_{i=1}^n \widetilde{e}_{iz}\overline{x}_{i,k} = \sum\nolimits_{i=1}^n \widetilde{e}_{iz}\frac{\sum_{p=1}^M \sum_{q=1}^M x_{i,k}^{<p,q>}}{M^2} \le R_t, \tag{11}$$

which means the output solution is feasible for ROCK.

Next, define

$$E(\boldsymbol{\pi}) \triangleq \sum\nolimits_{j=1}^m \min\{\sum\nolimits_{i=1}^n P_{ij}\sum_{k=1}^j x_{i,k}(T_k - T_{k-1}), E_j\}$$

as the optimization objective value for solution $\boldsymbol{\pi}$. Define $\boldsymbol{\pi}_1 \succeq \boldsymbol{\pi}_2$ if $\mathbf{x}_{i,1} \succeq \mathbf{x}_{i,2}$ for $i = 1, \ldots, n$, which in turn means $x_{i,1}^k \ge x_{i,2}^k$ for $k = 1, \ldots, m$. Apparently we always have $\boldsymbol{\pi} \succeq \mathbf{0}$. Further, to assist the following analysis, we define the *effective charging energy share* to measure the contribution of a portion of a solution $\boldsymbol{\pi}$, say $\boldsymbol{\pi}'$ where $\boldsymbol{\pi} \succeq \boldsymbol{\pi}'$, as

$$\begin{aligned}\widehat{E}(\boldsymbol{\pi}', \boldsymbol{\pi}) \triangleq \sum\nolimits_{j=1}^m \Bigg\{ &\frac{\sum_{i=1}^n P_{ij}\sum_{k=1}^j x'_{i,k}(T_k - T_{k-1})}{\sum_{i=1}^n P_{ij}\sum_{k=1}^j x_{i,k}(T_k - T_{k-1})} \\ &\times \min\{\sum\nolimits_{i=1}^n P_{ij}\sum_{k=1}^j x_{i,k}(T_k - T_{k-1}), E_j\}\Bigg\}. \end{aligned} \tag{12}$$

We name $\boldsymbol{\pi}$ *full solution* and $\boldsymbol{\pi}'$ *partial solution* for convenience. It is easy to verify that the above function has the following properties:

($\boldsymbol{\mathcal{P}1}$): $\widehat{E}(\boldsymbol{\pi}', \boldsymbol{\pi}) \le E(\boldsymbol{\pi}')$;

($\boldsymbol{\mathcal{P}2}$): $\widehat{E}(\boldsymbol{\pi}, \boldsymbol{\pi}) = E(\boldsymbol{\pi})$;

($\boldsymbol{\mathcal{P}3}$): $\widehat{E}(\boldsymbol{\pi}_1, \boldsymbol{\pi}) + \widehat{E}(\boldsymbol{\pi}_2, \boldsymbol{\pi}) = \widehat{E}(\boldsymbol{\pi}_1 + \boldsymbol{\pi}_2, \boldsymbol{\pi})$ if $\boldsymbol{\pi} \succeq \boldsymbol{\pi}_1 + \boldsymbol{\pi}_2$;

($\boldsymbol{\mathcal{P}4}$): $\sum_{i=1}^n \widehat{E}(\boldsymbol{\pi}_i, \boldsymbol{\pi}) = kE(\boldsymbol{\pi}) + \widehat{E}(\sum_{i=1}^n \boldsymbol{\pi}_i - k\boldsymbol{\pi}, \boldsymbol{\pi})$ if $\boldsymbol{\pi} \succeq \sum_{i=1}^n \boldsymbol{\pi}_i - k\boldsymbol{\pi} \succeq \mathbf{0}$ where k is a positive integer.

Suppose the obtained solution combined from all optimal solutions for inner rectangles of all $(M-1)$-*Cluster*s at Step 10 in Algorithm 2 for the turn-off policy $< p, q >$ is $\boldsymbol{\pi}_I^{<p,q>}$. Recall that the solution obtained at Step 11 in Algorithm 2 with a "relaxed" EMR threshold $\frac{1}{1-\varepsilon/3}R_t$ is $\boldsymbol{\pi}_R^{<p,q>}$. Clearly, we have $E(\boldsymbol{\pi}_R^{<p,q>}) = E(\boldsymbol{\pi}_I^{<p,q>})$. As the final solution is $\boldsymbol{\pi}^{<p,q>}$, which is equal to $(1-\varepsilon/3)\boldsymbol{\pi}^{<p,q>}$ as shown at Step 12 in Algorithm 2, and it is easy to see that $E(\frac{1}{1+\varepsilon/3}\boldsymbol{\pi}) \ge \frac{1}{1+\varepsilon/3}E(\boldsymbol{\pi})$, we get

$$\begin{aligned} E(\boldsymbol{\pi}^{<p,q>}) &= E((1-\varepsilon/3)\boldsymbol{\pi}_R^{<p,q>}) \ge (1-\varepsilon/3)E(\boldsymbol{\pi}_R^{<p,q>}) \\ &\ge (1-\varepsilon/3)E(\boldsymbol{\pi}_I^{<p,q>}). \end{aligned} \tag{13}$$

Suppose the optimal solution for problem **P3** (or **P4**) is $\widetilde{\boldsymbol{\pi}}^*$. Assume there are in total K number of M-*Cluster*s after the area partition. Define the partial solution $\widetilde{\boldsymbol{\pi}}^*_{ijk}$ as the solution for which each charger located in the i-th row and j-th column in the k-th M-*Cluster* sets its adjusting factor as that in the optimal solution $\widetilde{\boldsymbol{\pi}}^*$ while the other chargers are switched off. Moreover, suppose the aggregate effective charging energy share in the optimal solution $\widetilde{\boldsymbol{\pi}}^*$ for the chargers that are outside all the inner rectangles and are switched off for the turn-off policy $< p, q >$ $\boldsymbol{\pi}^{<p,q>}$ is $\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}$. As $\boldsymbol{\pi}_I^{<p,q>}$ is optimal for all inner rectangles of all $(M-1)$-*Cluster*s for the turn-off policy $< p, q >$, it should be better than or equal to the solution obtained by subtracting $\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}$ from the optimal solution $\widetilde{\boldsymbol{\pi}}^*$ as the latter is clearly feasible to the same setting as that of $\boldsymbol{\pi}_I^{<p,q>}$. Thus we get

$$E(\boldsymbol{\pi}_I^{<p,q>}) \ge E(\widetilde{\boldsymbol{\pi}}^* - \widetilde{\boldsymbol{\pi}}^{*<p,q>^-})$$

$$\overset{by\ (\mathcal{P}1)}{\geq} \widehat{E}(\widetilde{\boldsymbol{\pi}}^* - \widetilde{\boldsymbol{\pi}}^{*<p,q>^-}, \widetilde{\boldsymbol{\pi}}^*)$$
$$\overset{by\ (\mathcal{P}3)}{=} \widehat{E}(\widetilde{\boldsymbol{\pi}}^*, \widetilde{\boldsymbol{\pi}}^*) - \widehat{E}(\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}, \widetilde{\boldsymbol{\pi}}^*)$$
$$\overset{by\ (\mathcal{P}2)}{=} E(\widetilde{\boldsymbol{\pi}}^*) - \widehat{E}(\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}, \widetilde{\boldsymbol{\pi}}^*). \tag{14}$$

Moreover, it is easy to check that

$$\begin{aligned}
&\widetilde{\boldsymbol{\pi}}^{*<p,q>^-} \\
=&\sum_{k=1}^{K}(\sum_{j=1}^{M}\sum_{i=(p-1) \bmod M}^{(p+1) \bmod M} \widetilde{\boldsymbol{\pi}}^*_{ijk} + \sum_{j=(q-1) \bmod M}^{(q+1) \bmod M}\sum_{i=1}^{M}\widetilde{\boldsymbol{\pi}}^*_{ijk} \\
&-\sum_{j=(q-1) \bmod M}^{(q+1) \bmod M}\sum_{i=(p-1) \bmod M}^{(p+1) \bmod M}\widetilde{\boldsymbol{\pi}}^*_{pqk}).
\end{aligned} \tag{15}$$

Therefore, we get

$$\begin{aligned}
&\sum_{p=1}^{M}\sum_{q=1}^{M}\widetilde{\boldsymbol{\pi}}^{*<p,q>^-} \\
=&\sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{k=1}^{K}(\sum_{j=1}^{M}\sum_{i=(p-1) \bmod M}^{(p+1) \bmod M} \widetilde{\boldsymbol{\pi}}^*_{ijk} + \sum_{j=(q-1) \bmod M}^{(q+1) \bmod M}\sum_{i=1}^{M}\widetilde{\boldsymbol{\pi}}^*_{ijk} \\
&-\sum_{j=(q-1) \bmod M}^{(q+1) \bmod M}\sum_{i=(p-1) \bmod M}^{(p+1) \bmod M}\widetilde{\boldsymbol{\pi}}^*_{pqk}) \\
=&3\sum_{q=1}^{M}(\sum_{k=1}^{K}\sum_{j=1}^{M}\sum_{i=1}^{M}\widetilde{\boldsymbol{\pi}}^*_{ijk}) + 3\sum_{p=1}^{M}(\sum_{k=1}^{K}\sum_{j=1}^{M}\sum_{i=1}^{M}\widetilde{\boldsymbol{\pi}}^*_{ijk}) \\
&-9\sum_{p=1}^{M}\sum_{q=1}^{M}\sum_{k=1}^{K}\widetilde{\boldsymbol{\pi}}^*_{pqk} \\
=&(6M-9)\widetilde{\boldsymbol{\pi}}^*.
\end{aligned} \tag{16}$$

Combining (14) and (16), we get

$$\begin{aligned}
&\sum_{p=1}^{M}\sum_{q=1}^{M}E(\boldsymbol{\pi}_I^{<p,q>}) \\
\geq&\sum_{p=1}^{M}\sum_{q=1}^{M}[E(\widetilde{\boldsymbol{\pi}}^*) - \widehat{E}(\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}, \widetilde{\boldsymbol{\pi}}^*)] \\
=&\sum_{p=1}^{M}\sum_{q=1}^{M}E(\widetilde{\boldsymbol{\pi}}^*) - \sum_{p=1}^{M}\sum_{q=1}^{M}\widehat{E}(\widetilde{\boldsymbol{\pi}}^{*<p,q>^-}, \widetilde{\boldsymbol{\pi}}^*) \\
\overset{by\ (\mathcal{P}4)}{=}& M^2E(\widetilde{\boldsymbol{\pi}}^*) - [(6M-9)E(\widetilde{\boldsymbol{\pi}}^*) - \widehat{E}(\mathbf{0}, \widetilde{\boldsymbol{\pi}}^*)] \\
=&(M-3)^2E(\widetilde{\boldsymbol{\pi}}^*).
\end{aligned} \tag{17}$$

Besides, it is easy to see that the function $\min\{\sum_{i=1}^{n} P_{ij}\sum_{k=1}^{j} x_{i,k}(T_k - T_{k-1}), E_j\}$ is concave in terms of $x_{i,k}$ ($i = 1, \dots, n$; $k = 1, \dots, j$), and therefore $E(\boldsymbol{\pi}) = \sum_{j=1}^{m}\min\{\sum_{i=1}^{n} P_{ij}\sum_{k=1}^{j} x_{i,k}(T_k - T_{k-1}), E_j\}$ is also concave in terms of $x_{i,k}$ ($i = 1, \dots, n$; $j = 1, \dots, m$). Using Jensen's inequality for the concave functions [35], for the output solution of Algorithm 2, *i.e.* $\overline{\boldsymbol{\pi}}$, we get

$$\begin{aligned}
&E(\overline{\boldsymbol{\pi}}) \\
=&E\left(\frac{\sum_{p=1}^{M}\sum_{q=1}^{M}\boldsymbol{\pi}^{<p,q>}}{M^2}\right) \\
=&E\left(\frac{(M-1)^2}{M^2}\frac{\sum_{\substack{p,q\in[1,M] \\ <p,q>\neq<M,M>}}\boldsymbol{\pi}^{<p,q>}}{(M-1)^2} + \frac{1}{M^2}\boldsymbol{\pi}^{<M,M>}\right) \\
\geq&\frac{(M-1)^2}{M^2}E\left(\frac{\sum_{\substack{p,q\in[1,M] \\ <p,q>\neq<M,M>}}\boldsymbol{\pi}^{<p,q>}}{(M-1)^2}\right) + \frac{1}{M^2}E(\boldsymbol{\pi}^{<M,M>}) \\
\geq&\dots\dots \\
\geq&\frac{1}{M^2}\sum_{p=1}^{M}\sum_{q=1}^{M}E(\boldsymbol{\pi}^{<p,q>}) \\
\geq&(1-\varepsilon/3)\frac{1}{M^2}\sum_{p=1}^{M}\sum_{q=1}^{M}E(\boldsymbol{\pi}_I^{<p,q>}) && (\because\ (13)) \\
\geq&(1-\varepsilon/3)\frac{(M-3)^2}{M^2}E(\widetilde{\boldsymbol{\pi}}^*) && (\because\ (17)) \\
=&(1-\varepsilon/3)^2E(\widetilde{\boldsymbol{\pi}}^*). && (\because\ M = \lceil\frac{3}{1-\sqrt{1-\varepsilon/3}}\rceil)
\end{aligned} \tag{18}$$

Furthermore, we can prove that the solution regularization technique also applies to the original problem **P1**. Here we omit the details to save space. Consequently, suppose $\boldsymbol{\pi}^*$ is the optimal solution to **P1**. Then we get

$$C_1\sum\nolimits_{i=1}^{n}P(d(s_i,p))x^*_{i,k} \leq R_t \tag{19}$$

where $p \in \Omega$ and $k = 1, \dots, m$.

Given that Algorithm 2 sets the error threshold for EMR approximation as $\varepsilon/3$ at Step 1, as per (5), the approximated EMR at $p \in \mathcal{A}_z$ after area discretization satisfies

$$\sum_{i=1}^{n}\widetilde{e}_{iz}x^*_{i,k} \leq (1+\varepsilon/3)C_1\sum_{i=1}^{n}P(d(s_i,p))x^*_{i,k}. \tag{20}$$

where $z = 1, \dots, Z$ and $k = 1, \dots, m$. Combining (19) and (20) we get

$$\sum_{i=1}^{n}\widetilde{e}_{iz}\frac{x^*_{i,k}}{(1+\varepsilon/3)} \leq C_1\sum_{i=1}^{n}P(d(s_i,p))x^*_{i,k} \leq R_t. \tag{21}$$

This indicates that $\frac{1}{1+\varepsilon/3}\boldsymbol{\pi}^*$ is a feasible solution for the problem **P3**. As $\widetilde{\boldsymbol{\pi}}^*$ is the optimal solution to **P3**, we get $E(\widetilde{\boldsymbol{\pi}}^*) \geq E(\frac{1}{1+\varepsilon/3}\boldsymbol{\pi}^*)$. Moreover, it is easy to check that $E(\frac{1}{1+\varepsilon/3}\boldsymbol{\pi}^*) \geq \frac{1}{1+\varepsilon/3}E(\boldsymbol{\pi}^*)$. By (18), we obtain

$$\begin{aligned}
E(\overline{\boldsymbol{\pi}}) &\geq (1-\varepsilon/3)^2E(\widetilde{\boldsymbol{\pi}}^*) \geq (1-\varepsilon/3)^2\frac{1}{1+\varepsilon/3}E(\boldsymbol{\pi}^*) \\
&\geq (1-\varepsilon)E(\boldsymbol{\pi}^*),
\end{aligned} \tag{22}$$

which indicates Algorithm 2 achieves $(1-\varepsilon)$ approximation ratio in terms of overall effective charging energy.

Due to space limit, we omit further analysis and refer readers to [30] for more details. □

7 SIMULATION RESULTS

In this section, we conduct simulations to verify the performance of the proposed algorithms.

7.1 Evaluation Setup

If not stated otherwise, we use the following setup throughout our simulations. The related parameters in the charging model are $\alpha = 10^5$, $\beta = 40$, $D = 20\,m$, $C_1 = 0.01$. We uniformly scatter 1000 chargers and 2000 devices in a $2000\,m \times 2000\,m$ square area. The required charging energy and charging deadline for devices are randomly generated in the ranges of $[0\,J\ \ 100\,J]$ and $[0\,s\ \ 10\,s]$, respectively. Moreover, each obtained data point stands for the average result for 100 randomly generated topologies.

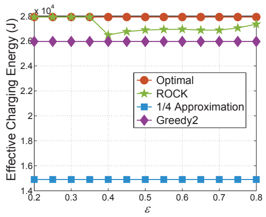


Figure 5: Effective charging energy vs. ε

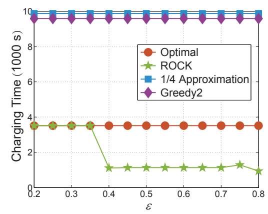


Figure 6: Charging time vs. ε

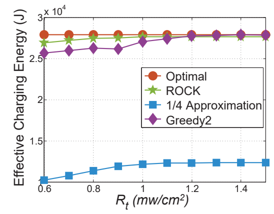


Figure 7: Effective charging energy vs. R_t

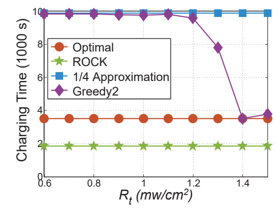


Figure 8: Charging time vs. R_t

7.2 Baseline Setup

We compare the centralized algorithm to ROCK with Greedy1, Greedy2, and Conopt, please see [30] for more details of them. Moreover, we compare the distributed algorithm to ROCK with three other algorithms, denoted by "Optimal", "1/4 Approximation", and "Greedy2", respectively, in figures. Optimal (or Greedy2) let all chargers send their information along with that of their covered devices to a sink node, which then executes our proposed optimal centralized algorithm to ROCK (or Greedy2), and then disseminates the results to all chargers. In contrast, 1/4 Approximation divides the whole area into multiple $2D \times 2D$ subareas and executes our centralized algorithm to ROCK in each subarea, and then cuts down the obtained adjusting factors of all chargers to 1/4.

7.3 Distributed Algorithm Evaluation

7.3.1 Impact of error threshold ε. *Our simulation results show that on average our algorithm to ROCK outperforms 1/4 Approximation and Greedy2 by* 82.6% *and* 4.83%, *respectively, in terms of effective charging energy, and* 597.8% *and* 577.5%, *respectively, in terms of charging time, and its achieved charging time is* 47.1% *smaller than that of Optimal.* Figures 7 and 8 show that the effective charging energy and charging time for Optimal, 1/4 Approximation, and Greedy2 remain unchanged as ε increases because they use gridding method with fixed granularity to partition area and approximate EMR, and that for our algorithm generally drops. Nevertheless, our algorithm still achieves on average 97.5% and at least 94.9% of Optimal in terms of effective charging energy even when ε is no less than 0.2, which supports Theorem 6.1.

7.3.2 Impact of EMR threshold R_t. *Our simulation results show that on average our algorithm to ROCK outperforms 1/4 Approximation and Greedy2 by* 133.9% *and* 2.0%, *respectively, in terms of effective charging energy, and* 432.3% *and* 350.1%, *respectively, in terms of charging time, and its achieved effective charging energy is larger than* $(1-\varepsilon)$ *of Optimal, and achieved charging time is* 47.7% *smaller than that of Optimal.* Figure 5 shows that the effective charging energy for all but Optimal slightly rises when R_t increases, because higher EMR threshold allows higher adjusting factors of chargers. Optimal keeps constant because all devices have already achieved their required charging energy. Especially, ROCK achieves at least 96.4% effective charging energy of Optimal given that $\varepsilon = 0.8$. Figure 6 shows that charging time for all but Greedy2 remains unchanged.

7.3.3 Impact of network size on delay. *Our simulation results show that the network delay of our algorithm to ROCK approaches a constant value of* 914 *as the networks size increases, and outperforms both Optimal and Greedy2 by* 74.4%. Figure 9 shows that the network delay for both Optimal and Greedy2 increases proportionally to the network size because the two algorithms require network-wide information collection and dissemination. In contrast, the network delay for either ROCK and 1/4 Approximation first increases and then approaches to a constant value. Particularly, 1/4 Approximation has even shorter delay as it incurs rather low communication cost inside $2D \times 2D$ small subareas.

8 FIELD EXPERIMENTS

In this section, we conduct field experiments to evaluate the performance of the ROCK algorithm. Figure 10 shows our testbed which consists of eight TX91501 power transmitters produced by Powercast [36] and marked by red numbers, and four rechargeable sensor nodes marked by blue numbers, as well as an AP connecting to a laptop for reporting the collected data from sensor nodes. The eight chargers are deployed on the vertices and middle points of edges of a $2.4\,m \times 2.4\,m$ square area with orientation angles $296.56°$, $296.56°$, $243.44°$, $26.56°$, $153.44°$, $63.44°$, $116.56°$ and $116.56°$, respectively. As the power of the chargers is not adjustable, we put a piece of copper foil tape with proper length, width, position, and bending angle in front of chargers so that the received power and EMR at locations further than the tape are cut down to desired levels. The four sensor nodes are placed at points $(0.7\,1.5)$, $(1.2\,1.2)$, $(1.6\,1.2)$, and $(1.8\,0.5)$ by taking the central point of the area as origin, and with orientation angles $220°$, $180°$, $70°$, and $90°$, respectively.

We set the effective required charging energy as random numbers in [0 50], *i.e.*, $24.92\,J$, $32.75\,J$, $8.13\,J$, and $5.95\,J$, and the charging deadline for sensor nodes as $5000\,s$, $2000\,s$, $2000\,s$, and $5000\,s$. Our goal is to minimize the charging time needed to achieve effective charging energy of $30\,J$. The error bound ε is set to 0.05. Figure 11 shows that the required charging time for ROCK is only 70.6 %, 27.2 %, and 85.0 % of that of Greedy1, Greedy2 and Conopt (see [30]) respectively when $R_t = 105\,\mu W/cm^2$, and becomes 71.2 %, 27.9 %, and 75.7 % when $R_t = 125\,\mu W/cm^2$. Figure 12 shows the EMR distribution on the considered field measured by a RF field strength meter when $R_t = 115$ for our ROCK solution which contains only one round. We can observe that even the maximum EMR $113.2\,\mu W/cm^2$ does not exceed $115\,\mu W/cm^2$, which means the EMR safety is achieved.

9 CONCLUSION

The key novelty of this paper is on proposing the first scheme for radiation constrained scheduling for wireless charging

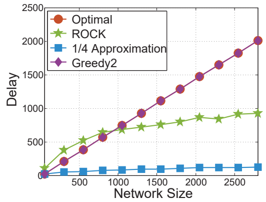


Figure 9: Delay

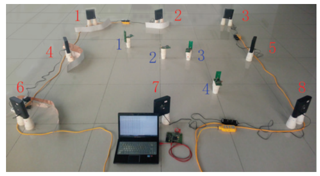


Figure 10: Testbed

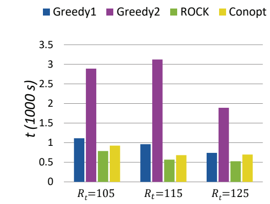


Figure 11: Charging time for energy of $30\,J$

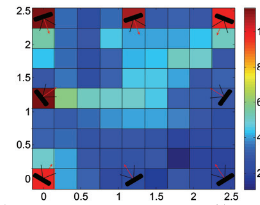


Figure 12: EMR distribution for $R_t = 115\,\mu W/cm^2$

tasks. The key contribution of this paper is proposing an optimal centralized algorithm and a distributed and scalable algorithm that achieves at least $(1-\varepsilon)$ of optimal effective charging energy and shorter charging time than the optimal solution, and conducting both simulations and field experiments for evaluation. The key technical depth of this paper is in making the nonlinear relaxed version of ROCK linear by presenting the approaches of area discretization and solution regularization; developing a linear programming based greedy test algorithm to optimally solve ROCK; and proposing a distributed algorithm scalable with network size based on an area partition scheme, and bounding its performance by proposing the area-scaling and EMR-scaling approaches. Our evaluation results show that our distributed algorithm achieves at least 94.9% of the optimal effective charging energy and requires only 47.1% of the optimal charging time when $\varepsilon \geq 0.2$, outperforms the other algorithms by at least 2.0% and 350.1% in terms of effective charging energy and charging time, respectively.

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